Name:
Course: CAP 4601
Semester: Summer 2013
Assignment: Assignment 2
Date: 03 JUN 2013

Complete the following written problems:

1. (10 Points) Given that a loaded coin has the following probability for coming up heads: $P(\text{Loaded Coin} = \text{heads}) = 0.25$. What is the probability that the loaded coin will come up tails? In other words, what is $P(\text{Loaded Coin} = \text{tails})$?

$$\begin{aligned} P(\text{Loaded Coin} = \text{tails}) &= 1 - P(\text{Loaded Coin} = \text{heads}) \\ &= 1 - 0.25 \\ &= 0.75 \end{aligned}$$

2. (20 Points) Given loaded coin tosses are independent events and that a different loaded coin has the following probability for coming up heads twice in a row: $P(\text{Loaded Coin} = \text{heads}, \text{Loaded Coin} = \text{heads}) = 0.16$. What is the probability that the loaded coin will come up tails twice in a row? In other words, what is $P(\text{Loaded Coin} = \text{tails}, \text{Loaded Coin} = \text{tails})$?

Since consecutive coin tosses do not depend on each other, then we have Independence such that:

$$
\begin{aligned}
P(\text{Loaded Coin} = \text{heads}, \text{Loaded Coin} = \text{heads}) &= 0.16 \\
P(\text{Loaded Coin} = \text{heads})\,P(\text{Loaded Coin} = \text{heads}) &= 0.16 \\
\left(P(\text{Loaded Coin} = \text{heads})\right)^2 &= 0.16 \\
\sqrt{\left(P(\text{Loaded Coin} = \text{heads})\right)^2} &= \underbrace{\pm\sqrt{0.16}}_{\substack{\text{We only care} \\ \text{about positive} \\ \text{values in} \\ \text{probability}}} \\
P(\text{Loaded Coin} = \text{heads}) &= 0.4
\end{aligned}
$$

$$
\begin{aligned}
P(\text{Loaded Coin} = \text{tails}) &= 1 - P(\text{Loaded Coin} = \text{heads}) \\
&= 1 - 0.4 \\
&= 0.6
\end{aligned}
$$

$$
\begin{aligned}
P(\text{Loaded Coin} = \text{tails}, \text{Loaded Coin} = \text{tails}) &= \underbrace{P(\text{Loaded Coin} = \text{tails})\,P(\text{Loaded Coin} = \text{tails})}_{\text{Because of Independence}} \\
&= (0.6)(0.6) \\
&= 0.36
\end{aligned}
$$

3. (40 Points) Given a fair coin with $P(\text{Fair Coin} = \text{heads}) = 0.5$ and a loaded coin with $P(\text{Loaded Coin} = \text{heads}) = 1$, if we pick a coin at random (i.e. $P(\text{Choice} = \text{Fair Coin}) = P(\text{Choice} = \text{Loaded Coin}) = 0.5$) and flip it, what is the probability that it is the loaded coin given that we observe heads? In other words, what is $P(\text{Choice} = \text{Loaded Coin} \mid \text{Coin} = \text{heads})$?

$$P(\text{Fair Coin} = \text{heads}) = 0.5$$

$$\begin{aligned} P(\text{Fair Coin} = \text{tails}) &= 1 - P(\text{Fair Coin} = \text{heads}) \\ &= 1 - 0.5 \\ &= 0.5 \end{aligned}$$

$$P(\text{Loaded Coin} = \text{heads}) = 1$$

$$\begin{aligned} P(\text{Loaded Coin} = \text{tails}) &= 1 - P(\text{Loaded Coin} = \text{heads}) \\ &= 1 - 1 \\ &= 0 \end{aligned}$$

$$\begin{aligned} P(\text{Coin} = \text{heads} \mid \text{Choice} = \text{Loaded Coin}) &= P(\text{Loaded Coin} = \text{heads}) \\ &= 1 \end{aligned}$$

$$\begin{aligned} P(\text{Coin} = \text{heads} \mid \text{Choice} = \text{Fair Coin}) &= P(\text{Fair Coin} = \text{heads}) \\ &= 0.5 \end{aligned}$$

$$P\left(\text{Choice} = \text{Loaded Coin} \middle| \text{Coin} = \text{heads}\right)$$

$$= \frac{P\left(\text{Coin} = \text{heads} \middle| \text{Choice} = \text{Loaded Coin}\right) P\left(\text{Choice} = \text{Loaded Coin}\right)}{P\left(\text{Coin} = \text{heads}\right)}$$

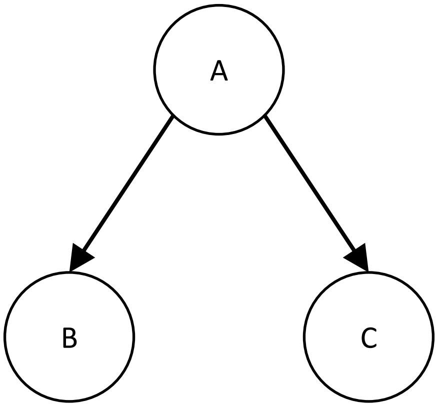
$$= \frac{P\left(\text{Coin} = \text{heads} \middle| \text{Choice} = \text{Loaded Coin}\right) P\left(\text{Choice} = \text{Loaded Coin}\right)}{\left(\begin{array}{l} P\left(\text{Coin} = \text{heads} \middle| \text{Choice} = \text{Loaded Coin}\right) P\left(\text{Choice} = \text{Loaded Coin}\right) \\ + \\ P\left(\text{Coin} = \text{heads} \middle| \text{Choice} = \text{Fair Coin}\right) P\left(\text{Choice} = \text{Fair Coin}\right) \end{array}\right)}$$

$$= \frac{(1)(0.5)}{\left(\begin{array}{l} (1)(0.5) \\ + \\ (0.5)(0.5) \end{array}\right)}$$

$$= \frac{2}{3}$$

$$\approx 0.6666666666666667$$

4. (40 Points) Given the following Bayes Network:



With the following probabilities:

$$P(A) = 0.5$$
$$P(B|A) = 0.2$$
$$P(B|\neg A) = 0.2$$
$$P(C|A) = 0.8$$
$$P(C|\neg A) = 0.4$$

- What is $P(B|C)$?

Since $P(B|A) = 0.2$ and $P(B|\neg A) = 0.2$ (i.e. they're both 0.2), then B does not depend on A; therefore, we have:

$$P(B|A) = P(B) \text{ and } P(B|\neg A) = P(B)$$

Moreover, since B doesn't depend on A, then B doesn't depend on C either. Therefore, we have:

$$P(B|C) = P(B) = 0.2$$

- What is $P(C|B)$?

Because of the Independence of C and B, we have:

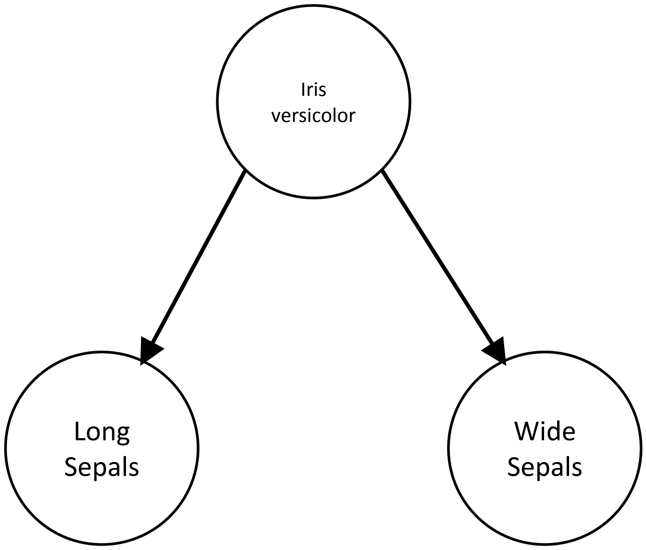
$$\begin{aligned} P(C|B) &= P(C) \\ &= P(C|A)P(A) + P(C|\neg A)P(\neg A) \\ &= (0.8)(0.5) + (0.4)(0.5) \\ &= 0.6 \end{aligned}$$

5. (130 Points) Using the data below, construct a Naïve Bayesian Network that does **NOT** use Laplacian Smoothing to predict that an Iris is an Iris versicolor based on if its sepals are long or wide.

Note: Sepals are the green petal-like objects surrounding a flower.

Long Sepals	**Wide Sepals**	**Iris versicolor**
false	true	false
false	false	false
false	true	false
false	true	false
false	true	false
false	true	false
false	true	false
false	true	false
false	false	false
false	true	false
true	true	true
true	true	true
true	true	true
true	false	true
true	false	true
true	false	true
true	true	true
false	false	true
true	false	true
false	false	true

Do the following:
- Draw the graph of the Naïve Bayesian Network.



- Given the data above, answer the following questions:

a. What is the probability that an Iris is an Iris versicolor? In other words, what is $P(\text{Iris versicolor} = \text{true})$?

Long Sepals	**Wide Sepals**	**Iris versicolor**
false	true	false
false	false	false
false	true	false
false	true	false
false	true	false
false	true	false
false	true	false
false	true	false
false	false	false
false	true	false
true	true	true
true	true	true
true	true	true
true	false	true
true	false	true
true	false	true
true	true	true
false	false	true
true	false	true
false	false	true

$$P(\text{Iris versicolor} = \text{true}) = \frac{10}{20} = \frac{1}{2} = 0.5$$

b. Given an Iris versicolor, what is the probability that its sepals are long? In other words, what is $P(\text{Long Sepals} = \text{true} \mid \text{Iris versicolor} = \text{true})$?

Long Sepals	**Wide Sepals**	**Iris versicolor**
false	true	false
false	false	false
false	true	false
false	true	false
false	true	false
false	true	false
false	true	false
false	true	false
false	false	false
false	true	false
true	true	true
true	true	true
true	true	true
true	false	true
true	false	true
true	false	true
true	true	true
false	false	true
true	false	true
false	false	true

$$P(\text{Long Sepals} = \text{true} \mid \text{Iris versicolor} = \text{true}) = \frac{8}{10} = \frac{4}{5} = 0.8$$

c. Given an Iris versicolor, what is the probability that its sepals are wide? In other words, what is $P(\text{Wide Sepals} = \text{true} \mid \text{Iris versicolor} = \text{true})$?

Long Sepals	**Wide Sepals**	**Iris versicolor**
false	true	false
false	false	false
false	true	false
false	true	false
false	true	false
false	true	false
false	true	false
false	true	false
false	false	false
false	true	false
true	true	true
true	true	true
true	true	true
true	false	true
true	false	true
true	false	true
true	true	true
false	false	true
true	false	true
false	false	true

$$P(\text{Wide Sepals} = \text{true} \mid \text{Iris versicolor} = \text{true}) = \frac{4}{10} = \frac{2}{5} = 0.4$$

d. Given an Iris versicolor, what is the probability that its sepals are both long and wide? In other words, what is $P(\text{Long Sepals} = \text{true}, \text{Wide Sepals} = \text{true} \mid \text{Iris versicolor} = \text{true})$?

Long Sepals	**Wide Sepals**	**Iris versicolor**
false	true	false
false	false	false
false	true	false
false	true	false
false	true	false
false	true	false
false	true	false
false	true	false
false	false	false
false	true	false
true	true	true
true	true	true
true	true	true
true	false	true
true	false	true
true	false	true
true	true	true
false	false	true
true	false	true
false	false	true

If we do not use the Naïve Bayes assumption of conditional independence, then we have:

$$P(\text{Long Sepals} = \text{true}, \text{Wide Sepals} = \text{true} \mid \text{Iris versicolor} = \text{true}) = \frac{4}{10} = \frac{2}{5} = 0.4$$

If we do use the conditional independence assumption, then we have:

$$\begin{aligned}
&P(\text{Long Sepals} = \text{true}, \text{Wide Sepals} = \text{true} \mid \text{Iris versicolor} = \text{true}) \\
&= P(\text{Long Sepals} = \text{true} \mid \text{Iris versicolor} = \text{true})\, P(\text{Wide Sepals} = \text{true} \mid \text{Iris versicolor} = \text{true}) \\
&= \left(\frac{4}{5}\right)\left(\frac{2}{5}\right) \\
&= \frac{8}{25} \\
&= 0.32
\end{aligned}$$

e. Given an Iris that is not an Iris versicolor, what is the probability that its sepals are both long and wide? In other words, what is
$P\left(\text{Long Sepals} = \text{true}, \text{Wide Sepals} = \text{true} \middle| \text{Iris versicolor} = \text{false}\right)$?

Long Sepals	Wide Sepals	Iris versicolor
false	true	false
false	false	false
false	true	false
false	true	false
false	true	false
false	true	false
false	true	false
false	true	false
false	false	false
false	true	false
true	true	true
true	true	true
true	true	true
true	false	true
true	false	true
true	false	true
true	true	true
false	false	true
true	false	true
false	false	true

If we do not use the Naïve Bayes assumption of conditional independence, then we have:

$$P\left(\text{Long Sepals} = \text{true}, \text{Wide Sepals} = \text{true} \middle| \text{Iris versicolor} = \text{false}\right) = \frac{0}{10} = 0$$

If we do use the conditional independence assumption, then we have:

$$\begin{aligned}
&P\left(\text{Long Sepals} = \text{true}, \text{Wide Sepals} = \text{true} \middle| \text{Iris versicolor} = \text{false}\right) \\
&= P\left(\text{Long Sepals} = \text{true} \middle| \text{Iris versicolor} = \text{false}\right) P\left(\text{Wide Sepals} = \text{true} \middle| \text{Iris versicolor} = \text{false}\right) \\
&= \left(\frac{0}{10}\right)\left(\frac{8}{10}\right) \\
&= (0)\left(\frac{4}{5}\right) \\
&= 0
\end{aligned}$$

f. Given an Iris with both long and wide sepals, what is the probability that it's an Iris versicolor? In other words, what is $P(\text{Iris versicolor} = \text{true} \mid \text{Long Sepals} = \text{true}, \text{Wide Sepals} = \text{true})$?

By Bayes Rule, we have:

$$
\begin{aligned}
&P(\text{Iris versicolor} = \text{true} \mid \text{Long Sepals} = \text{true}, \text{Wide Sepals} = \text{true}) \\
&= \frac{P(\text{Long Sepals} = \text{true}, \text{Wide Sepals} = \text{true} \mid \text{Iris versicolor} = \text{true})\, P(\text{Iris versicolor} = \text{true})}{P(\text{Long Sepals} = \text{true}, \text{Wide Sepals} = \text{true})} \\
&= \frac{P(\text{Long Sepals} = \text{true}, \text{Wide Sepals} = \text{true} \mid \text{Iris versicolor} = \text{true})\, P(\text{Iris versicolor} = \text{true})}{P(\text{Long Sepals} = \text{true}, \text{Wide Sepals} = \text{true})} \\
&= \frac{\overbrace{P(\text{Long Sepals} = \text{true} \mid \text{Iris versicolor} = \text{true})\, P(\text{Wide Sepals} = \text{true} \mid \text{Iris versicolor} = \text{true})}^{\text{Naive Bayes assumes conditional independence}}\, P(\text{Iris versicolor} = \text{true})}{P(\text{Long Sepals} = \text{true}, \text{Wide Sepals} = \text{true})}
\end{aligned}
$$

Total probability for $P(\text{Long Sepals} = \text{true}, \text{Wide Sepals} = \text{true})$ is the following:

$$
\begin{aligned}
&P(\text{Long Sepals} = \text{true}, \text{Wide Sepals} = \text{true}) \\
&= \left(\begin{array}{l}
P(\text{Long Sepals} = \text{true}, \text{Wide Sepals} = \text{true} \mid \text{Iris versicolor} = \text{true})\, P(\text{Iris versicolor} = \text{true}) \\
+ \\
P(\text{Long Sepals} = \text{true}, \text{Wide Sepals} = \text{true} \mid \text{Iris versicolor} = \text{false})\, P(\text{Iris versicolor} = \text{false})
\end{array}\right) \\
&= \left(\begin{array}{l}
\overbrace{P(\text{Long Sepals} = \text{true} \mid \text{Iris versicolor} = \text{true})\, P(\text{Wide Sepals} = \text{true} \mid \text{Iris versicolor} = \text{true})}^{\text{Naive Bayes assumes conditional independence.}}\, P(\text{Iris versicolor} = \text{true}) \\
+ \\
\underbrace{P(\text{Long Sepals} = \text{true} \mid \text{Iris versicolor} = \text{false})\, P(\text{Wide Sepals} = \text{true} \mid \text{Iris versicolor} = \text{false})}_{\text{Naive Bayes assumes conditional independence.}}\, P(\text{Iris versicolor} = \text{false})
\end{array}\right) \\
&= \left(\begin{array}{l}
P(\text{Long Sepals} = \text{true} \mid \text{Iris versicolor} = \text{true})\, P(\text{Wide Sepals} = \text{true} \mid \text{Iris versicolor} = \text{true})\, P(\text{Iris versicolor} = \text{true}) \\
+ \\
P(\text{Long Sepals} = \text{true} \mid \text{Iris versicolor} = \text{false})\, P(\text{Wide Sepals} = \text{true} \mid \text{Iris versicolor} = \text{false})\, P(\text{Iris versicolor} = \text{false})
\end{array}\right)
\end{aligned}
$$

Therefore, we have:

$$P(\text{Iris versicolor} = \text{true} | \text{Long Sepals} = \text{true}, \text{Wide Sepals} = \text{true})$$

$$= \frac{P(\text{Long Sepals} = \text{true} | \text{Iris versicolor} = \text{true}) P(\text{Wide Sepals} = \text{true} | \text{Iris versicolor} = \text{true}) P(\text{Iris versicolor} = \text{true})}{P(\text{Long Sepals} = \text{true}, \text{Wide Sepals} = \text{true})}$$

$$= \frac{P(\text{Long Sepals} = \text{true} | \text{Iris versicolor} = \text{true}) P(\text{Wide Sepals} = \text{true} | \text{Iris versicolor} = \text{true}) P(\text{Iris versicolor} = \text{true})}{\left(\begin{array}{l} P(\text{Long Sepals} = \text{true} | \text{Iris versicolor} = \text{true}) P(\text{Wide Sepals} = \text{true} | \text{Iris versicolor} = \text{true}) P(\text{Iris versicolor} = \text{true}) \\ + \\ P(\text{Long Sepals} = \text{true} | \text{Iris versicolor} = \text{false}) P(\text{Wide Sepals} = \text{true} | \text{Iris versicolor} = \text{false}) P(\text{Iris versicolor} = \text{false}) \end{array} \right)}$$

$$= \frac{\left(\frac{4}{5}\right)\left(\frac{2}{5}\right)\left(\frac{1}{2}\right)}{\left(\begin{array}{l} \left(\frac{4}{5}\right)\left(\frac{2}{5}\right)\left(\frac{1}{2}\right) \\ + \\ (0)\left(\frac{4}{5}\right)\left(\frac{1}{2}\right) \end{array} \right)}$$

$$= \frac{\frac{8}{50}}{\frac{8}{50} + 0}$$

$$= 1$$

We can also see this from the table without using the Naïve Bayes assumption of conditional independence if we look at the values of Iris versicolor given $\text{Long sepals} = \text{true}$ and $\text{Wide sepal} = \text{true}$.

Long Sepals	**Wide Sepals**	**Iris versicolor**
false	true	false
false	false	false
false	true	false
false	true	false
false	true	false
false	true	false
false	true	false
false	true	false
false	false	false
false	true	false
true	true	true
true	true	true
true	true	true
true	false	true
true	false	true
true	false	true
true	true	true
false	false	true
true	false	true
false	false	true

6. (60 Points) Using the data from the previous problem, construct a Naïve Bayesian Network that **DOES** use Laplacian Smoothing to predict that an Iris is an Iris versicolor based on if its sepals are long or wide.

For Laplacian Smoothing, use $k = 1$.

NOTE: Everyone that turned in this portion will receive the full 60 points regardless of their answers.

- Given the data above, answer the following questions:

a. What is the probability that an Iris is an Iris versicolor? In other words, what is $P(\text{Iris versicolor} = \text{true})$?

$$P(\text{Iris versicolor} = \text{true}) = \frac{10+k}{20+k\cdot n} = \frac{10+1}{20+1\cdot 2} = \frac{11}{22} = \frac{1}{2} = 0.5$$

b. Given an Iris versicolor, what is the probability that its sepals are long? In other words, what is $P(\text{Long Sepals} = \text{true} \mid \text{Iris versicolor} = \text{true})$?

$$\begin{aligned} P(\text{Long Sepals} = \text{true} \mid \text{Iris versicolor} = \text{true}) &= \frac{8+k}{10+k\cdot n} \\ &= \frac{8+1}{10+1\cdot 2} \\ &= \frac{9}{12} \\ &= \frac{3}{4} \\ &= 0.75 \end{aligned}$$

c. Given an Iris versicolor, what is the probability that its sepals are wide? In other words, what is $P(\text{Wide Sepals} = \text{true} \mid \text{Iris versicolor} = \text{true})$?

$$\begin{aligned} P(\text{Wide Sepals} = \text{true} \mid \text{Iris versicolor} = \text{true}) &= \frac{4+k}{10+k\cdot n} \\ &= \frac{4+1}{10+1\cdot 2} \\ &= \frac{5}{12} \\ &\approx 0.4166666666666667 \end{aligned}$$

d. Given an Iris versicolor, what is the probability that its sepals are both long and wide? In other words, what is $P\left(\text{Long Sepals} = \text{true}, \text{Wide Sepals} = \text{true} \middle| \text{Iris versicolor} = \text{true}\right)$?

If we do not use the Naïve Bayes assumption of conditional independence, then we have:

$$\begin{aligned}
P\left(\text{Long Sepals} = \text{true}, \text{Wide Sepals} = \text{true} \middle| \text{Iris versicolor} = \text{true}\right) &= \frac{4+k}{10+k \cdot n} \\
&= \frac{4+1}{10+1\cdot 4} \\
&= \frac{5}{14} \\
&\approx 0.35714285714285715
\end{aligned}$$

If we do use the conditional independence assumption, then we have:

$$\begin{aligned}
&P\left(\text{Long Sepals} = \text{true}, \text{Wide Sepals} = \text{true} \middle| \text{Iris versicolor} = \text{true}\right) \\
&= P\left(\text{Long Sepals} = \text{true} \middle| \text{Iris versicolor} = \text{true}\right) P\left(\text{Wide Sepals} = \text{true} \middle| \text{Iris versicolor} = \text{true}\right) \\
&= \underbrace{\overset{\substack{\text{Already}\\\text{Laplacian}\\\text{Smoothed}}}{\left(\frac{3}{4}\right)} \overset{\substack{\text{Already}\\\text{Laplacian}\\\text{Smoothed}}}{\left(\frac{5}{12}\right)}}_{\substack{\text{Since everything is already}\\\text{Laplacian Smoothed, then}\\\text{we don't do it again.}}} \\
&= \frac{15}{48} \\
&= \frac{5}{16} \\
&= 0.3125
\end{aligned}$$

e. Given an Iris that is not an Iris versicolor, what is the probability that its sepals are both long and wide? In other words, what is
$P\left(\text{Long Sepals} = \text{true}, \text{Wide Sepals} = \text{true} \middle| \text{Iris versicolor} = \text{false}\right)$?

If we do not use the Naïve Bayes assumption of conditional independence, then we have:

$$\begin{aligned} P\left(\text{Long Sepals} = \text{true}, \text{Wide Sepals} = \text{true} \middle| \text{Iris versicolor} = \text{false}\right) &= \frac{0+k}{10+k \cdot n_{\text{Long Sepals \&Wide Sepals}}} \\ &= \frac{1}{10+1\cdot 4} \\ &= \frac{1}{14} \\ &\approx 0.07142857142857142 \end{aligned}$$

If we do use the conditional independence assumption, then we have:

$$\begin{aligned} &P\left(\text{Long Sepals} = \text{true}, \text{Wide Sepals} = \text{true} \middle| \text{Iris versicolor} = \text{false}\right) \\ &= P\left(\text{Long Sepals} = \text{true} \middle| \text{Iris versicolor} = \text{false}\right) P\left(\text{Wide Sepals} = \text{true} \middle| \text{Iris versicolor} = \text{false}\right) \\ &= \left(\frac{0+k}{10+k \cdot n_{\text{Long Sepals}}}\right)\left(\frac{8+k}{10+k \cdot n_{\text{Wide Sepals}}}\right) \\ &= \left(\frac{0+1}{10+1\cdot 2}\right)\left(\frac{8+1}{10+1\cdot 2}\right) \\ &= \left(\frac{1}{12}\right)\left(\frac{9}{12}\right) \\ &= \left(\frac{1}{12}\right)\left(\frac{3}{4}\right) \\ &= \frac{3}{48} \\ &= 0.0625 \end{aligned}$$

f. Given an Iris with both long and wide sepals, what is the probability that it's an Iris versicolor? In other words, what is $P(\text{Iris versicolor} = \text{true} \mid \text{Long Sepals} = \text{true}, \text{Wide Sepals} = \text{true})$?

$$
\begin{aligned}
&P(\text{Iris versicolor} = \text{true} \mid \text{Long Sepals} = \text{true}, \text{Wide Sepals} = \text{true}) \\
&= \frac{P(\text{Long Sepals} = \text{true} \mid \text{Iris versicolor} = \text{true})\,P(\text{Wide Sepals} = \text{true} \mid \text{Iris versicolor} = \text{true})\,P(\text{Iris versicolor} = \text{true})}{\left(\begin{array}{l} P(\text{Long Sepals} = \text{true} \mid \text{Iris versicolor} = \text{true})\,P(\text{Wide Sepals} = \text{true} \mid \text{Iris versicolor} = \text{true})\,P(\text{Iris versicolor} = \text{true}) \\ + \\ P(\text{Long Sepals} = \text{true} \mid \text{Iris versicolor} = \text{false})\,P(\text{Wide Sepals} = \text{true} \mid \text{Iris versicolor} = \text{false})\,P(\text{Iris versicolor} = \text{false}) \end{array}\right)} \\
&= \frac{\left(\frac{3}{4}\right)\left(\frac{5}{12}\right)\left(\frac{1}{2}\right)}{\left(\begin{array}{l} \left(\frac{3}{4}\right)\left(\frac{5}{12}\right)\left(\frac{1}{2}\right) \\ + \\ \left(\frac{1}{12}\right)\left(\frac{3}{4}\right)\left(\frac{1}{2}\right) \end{array}\right)} \\
&= \frac{\frac{5}{32}}{\left(\frac{5}{32} + \frac{1}{32}\right)} \\
&= \frac{5}{6} \\
&\approx 0.8333333333333334
\end{aligned}
$$

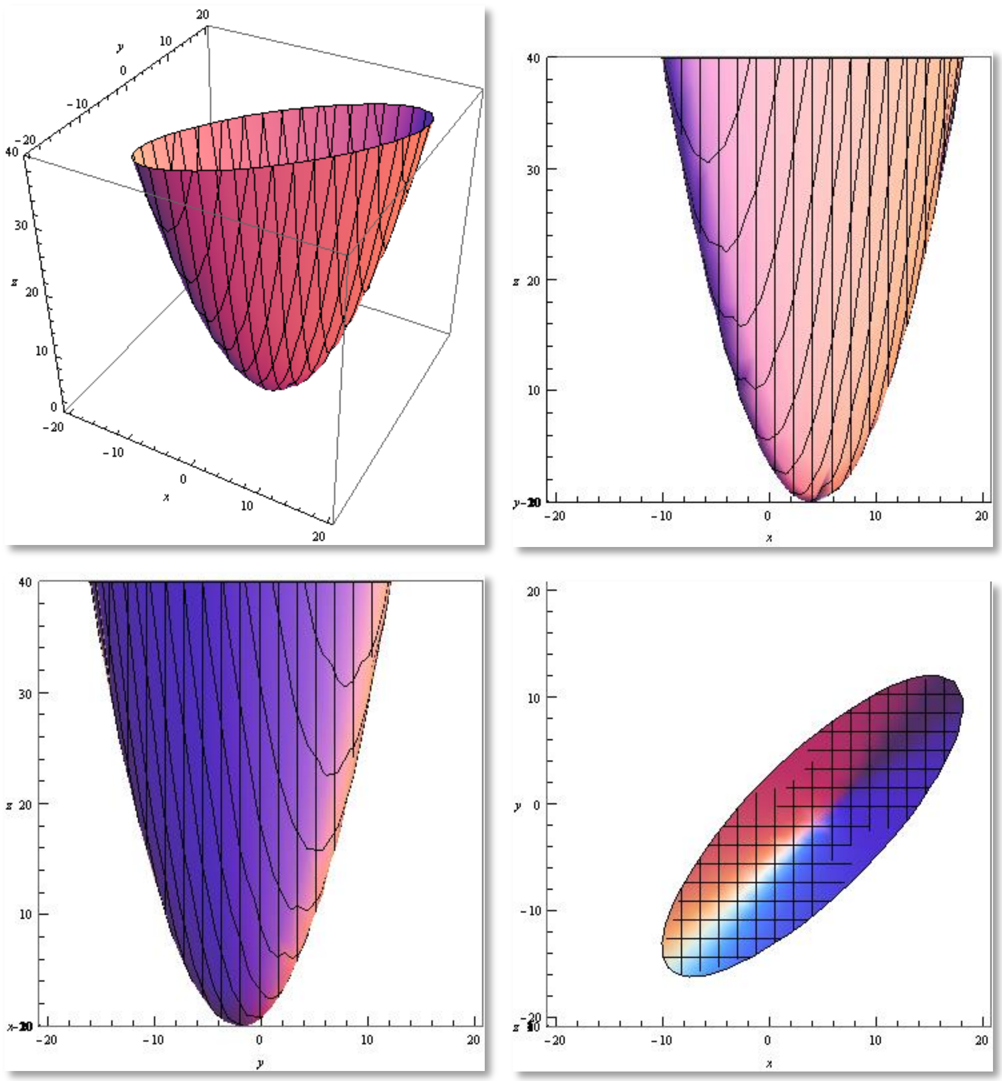
Complete the following programming problem on `linprog4.cs.fsu.edu`:

Download the ZIP file containing the directory structure and files for this programming problem: assignment_02.zip

1. (100 Points) Use the method of gradient descent to find the minimum of a function.

Given the function: $f(x,y)=\frac{5x^2}{9}-\frac{8xy}{9}-\frac{56x}{9}+\frac{5y^2}{9}+\frac{52y}{9}+\frac{164}{9}$

With the following plots:



Use the method of gradient descent to find the (x, y) value that produces the minimum $z = f(x, y)$ when we start gradient descent from $(0,0)$ and use the learning rate $\alpha = 0.1$.

The gradient descent update formula is: $\mathbf{x}_{n+1} \leftarrow \mathbf{x}_n - \alpha\nabla f(\mathbf{x}_n)$; therefore, it is $(x_{n+1}, y_{n+1}) \leftarrow (x_n, y_n) - \alpha\nabla f(x_n, y_n)$ for this problem.

Stop the gradient descent when either:
- The difference in consecutive $\mathbf{x}_i$ values is less than 0.000001. In other words, when $\|\mathbf{x}_{n+1} - \mathbf{x}_n\| < 0.000001$.
- The number of full gradient descent iterations exceeds 1024. In other words, don't do more than 1024 updates of gradient descent.

Use the following files:
- `print.hpp`: The file containing `operator <<` to print arrays and `std::vector`.
- `main.cpp`: The file for editing.
- `makefile`: The makefile for `linprog4.cs.fsu.edu`.

Do not make changes to the `makefile`. Only make changes to `main.cpp`.

Use `std::cout` to output information exactly in the following format:

```
1: ( 0, 0 )
2: ( 0.622222, -0.577778 )
3: ( 1.12395, -1.03605 )
.
.
.
```

Note: The ellipses above should not be included in your output. The ellipses represent the rest of your properly formatted output for this gradient descent problem.

After completing Assignment 02, create an `assignment_02_`*`lastname`*`.pdf` file for your written assignment and an `assignment_02_`*`lastname`*`.zip` file for your programming assignment (where *`lastname`* is your last name). Ensure that your `assignment_02_`*`lastname`*`.zip` retains the directory structure of the original zip file. In other words, ensure your zip file has the following directory structure:

- `/`
  - `gradient_descent/`
    - `print.hpp`
    - `main.cpp`
    - `makefile`

Upload both your `assignment_02_`*`lastname`*`.pdf` file for your written assignment and your `assignment_02_`*`lastname`*`.zip` file for your programming assignment to the Assignment 02 location on the [BlackBoard](https://campus.fsu.edu) site: https://campus.fsu.edu.