CNT 4504 Computer Networks March 27, 2025

Problem Set 4
Points are indicated for each problem. Total is 60 points.
Turn in before class: April 3, 2025

1. (10 points) Consider the sample space Ω consisting of the 16 pairs (x, y) where x and y are each one of 1, 2, 3, 4. Let the probabilities of each of the pairs with x = y be x/70, and let the probabilities of the other outcomes be 1/14.
   a. Show that this is in fact a probability measure.
   b. Consider the random variable X defined to be the value of the first component. Find the mean of X and the variance of X.
   c. Determine the probabilities of the following events:
      1. x = 1 (b) x = 2 or y = 1 (c) x > 1 and y > 2
      (d) $x + y \le 6$ (e) $| x - y | = 2$.

2. (20 pts) Find E(X), $E(X^2)$ and Var(X) if X is a random variable that is as below. Show your work.
   a. Exponentially distributed with parameter λ.
   b. Poisson distributed with parameter λ.
   c. Binomial with parameters (*n*,*p*).
   d. Geometric with p the probability of success.
   e. Uniformly distributed in the interval [0, 1].

3. (10 pts) Assume that packets sent on a long distance high-bandwidth communication channel are exponentially distributed with a mean length of 4000 bytes. What is the probability that a packet sent is longer than 10,000 bytes?

4. (10 pts) A college has 60% men and 40% women and it is known that 40% of the men and 60% of the women smoke. What is the probability that a student observed smoking a cigarette is a man?

5. This problem will not be graded. However, it is worth trying to solve it.
   Suppose that X and Y are geometrically distributed random variables with parameter p that is $P(X=x) = p(1-p)^x$ and similarly for Y. Find the distribution of Z= Min(X,Y).

6. (10 points) Consider a network of roads connecting cities, with being nodes of a graph and the roads being the edges. Assume that the probability that a road is open and traversable to be p independent of the other roads. For example, the probability that you can get from A to B over the following network would be p^2:

For each of the following two networks, find the probability that you can get from A to B.