COT4420 Theory of Computation November 9, 2024

Problem Set 5
Due November 18, 2024
No late extensions
Total of 85 points

1. (20 pts) Construct pda's that accept the following languages.
   (a) $L = \{a^n b^2 c^{n+m} : n, m \geq 0\}$
   (b) $L = \{a^n b^m : 2n \leq m \leq 3n\}$
   (c) $L = \{ w: n_a(w) + n_b(w) = n_c(w) + 2\}$
   (d) $L = \{ww : w \in \{a, b\}^+ \}$

2. (10 pts) Find a pda accepting by final state with no more than two control states for the language aa*b*a.

3. (10 pts) Show that L*, where $L = \{a^{2n} b^n, n \geq 0\}$, is a deterministic context free language by constructing the dpda for L*.

4. (15 pts) Consider the following language: $L = \{a^i b^j c^k : i > j, k > j \text{ and } j > 0\}$. Construct a pda for L if the language is context-free. If L is not a context-free language, prove this using the pumping lemma for context-free languages.

5. (15 pts) Let L_1 be a context free language and L_2 be a regular set. Show that there is an algorithm to determine whether or not L_1 and L_2 have an element in common.

6. (15 pts) Explain the following for context-free languages:
   (a) Are context-free languages closed under complement? Explain.
   (b) Are context-free languages closed under Kleene *? Explain.
   (c) Are deterministic context-free languages closed under intersection? Explain.

The following problems will not be graded and should not be turned in. You should try them to help you understand the material. The solutions will be covered in class.

7. We can define a restricted pda as one that can increase the length of the stack by at most one symbol in each move. Show that for every pda M there exists such a restricted pda M' such that L(M) = L(M').

8. Give an example of a deterministic context free language whose reverse is not deterministic.

9. Suppose you are allowed to define a *super* pda (spda) that extends a pda by using two stacks. Thus an spda move would pop or push symbols into either (or both) stack(s) based on the state, input and the symbols at the tops of each stack. Outline how you would use this spda to accept the language $L = \{wcw : w \in \{a, b\}^*\}$.

10. Show that there exists an algorithm to determine whether the language generated by a context-free grammar contains any words of length less than some given number *n*.