**Problem Set 4**
**Due November 12, 2024**
**Ten points each: total points are 80**

1. Find context-free grammars that will generate the following languages:
   (a) $L = \{a^n b^m : 2n \le m \le 3n, n \ge 0, m \ge 0\}$
   (b) $L = \{a^n b^m c^k : k = n + m, n \ge 0, m \ge 0\}$

2. The grammar below is ambiguous:
   $$E \to E + E \mid E * E \mid (E) \mid a \mid b$$
   (a) Extend this grammar to include subtraction (-) and exponentiation ( $\uparrow$ ).
   (b) Is this grammar ambiguous? If so give two different parse trees for some yield.
   (c) Construct an unambiguous grammar for your extended grammar. Make sure the precedence and associativity is as we would expect.

3. Consider the language $L = \{ a^n b^n c^m : n \ge 0, m \ge 0\} \cup \{ a^n b^m c^m : n \ge 0, m \ge 0\}$.
   (a) Find a context-free grammar for L.
   (b) Show that L is ambiguous.

4. Consider the grammar:
   (1) $E \to E + T \mid T$
   (2) $T \to T * F \mid F$
   (3) $F \to (E) \mid$ **id**

   (a) Give a rightmost derivation for the sentence w = **id** * (**id** + **id**) * **id + id**.
   (b) Give a left most derivation for the same string w.
   (c) The above grammar is left recursive. One way of eliminating left recursion is to replace productions of the form:
   $$A \to A\,\alpha \mid \beta,$$
   where $\beta$ does not begin with an A, by the productions:
   $$A \to \beta\, A'$$
   $$A' \to \alpha\, A' \mid \lambda.$$
   Transform the above grammar into one without left recursion using this technique.

5. Eliminate $\lambda$ productions from the following grammar.
   $$S \to AaB \mid aaB$$
   $$A \to \lambda$$
   $$B \to bbA \mid \lambda$$

6. Convert the following grammar to Greibach normal form:

$$S \rightarrow AA \mid a$$
$$A \rightarrow SS \mid b$$

7. Consider the grammar:
   (1) $S \rightarrow i\ C\ t\ S$
   (2) $S \rightarrow i\ C\ t\ S\ e\ S$
   (3) $S \rightarrow a$
   (4) $C \rightarrow b$

   where i, t, and, e stand for **if**, **then**, and **else**, and C and S for "conditional" and "statement."
   (a) Construct a rightmost derivation for the sentence w = i b t i b t a e a.
   (b) Show the corresponding parse tree for the above sentence.
   (c) Is the above grammar ambiguous? If so, prove it.

8. Eliminate unit productions from the following grammar. Show your work.

$$S \rightarrow XY$$
$$X \rightarrow A$$
$$A \rightarrow B \mid a$$
$$B \rightarrow b$$
$$Y \rightarrow T$$
$$T \rightarrow Y \mid c$$

The following problems will not be graded but will be covered in the solutions.

Transform the following grammar into Chomsky normal form.

$$S \rightarrow abAB$$
$$A \rightarrow bAB \mid \lambda$$
$$B \rightarrow BAa \mid A \mid \lambda$$

Consider the language $L = \{ww^R: w \in \{a,b\}^*\}$.
(a) Find a context-free grammar for L.
(b) Find a context-free grammar for the *complement* of L.