COT4420 Theory of Computation September 12, 2024

Problem Set 2
(total points are 90)
Due September 26, 2024

1. (10 pts) Let $\beta: X^+ \rightarrow Y$. Define the Nerode machine M_β for this β. Prove that the transition function and output function of M_β are well defined. Is the machine reduced? If so prove it, else give a counterexample. What is the significant of the Myhill-Nerode theorem?

2. (15pts) Prove that for two machines M and N, M indistinguishable from N does not imply that M has the same behavior as N. Hint: one way to prove this is to exhibit two machines, a two state machine M and a three state machine N, both over X = {0,1} and Y= {0,1} with this property.

3. (10 pts) Consider the machine M below. What is the input set X? What is the output set Y? What is the state set Q? Is this a Moore machine or a Mealy machine? Give the behavior β_q from each state q $\in$ Q, for the input sequence 32211. Give the behavior β_q from each state q for the input sequence 11322. Represent machine M using a transition graph.

Q \ X	1	2	3
q_1	q_2 / 1	q_4 / 1	q_2 / 1
q_2	q_3 / 0	q_2 / 1	q_4 / 0
q_3	q_2 / 0	q_4 / 0	q_1 / 1
q_4	q_3 / 1	q_1 / 0	q_3 / 1

MACHINE M

4. (15 pts) Let X = {*a, b*} and Y = {*1, 2, none, both*}. Let *x* be an input string. The number of *a*'s in the input string *x* is defined to be $N_a(x)$. Similarly, for $N_b(x)$. Construct a Mealy machine that, for an input string *x*, outputs a *1* if $N_a(x)$ (mod 5) is equal to 2, outputs a *2* if $N_b(x)$ (mod 3) is equal to 1, and outputs "*both*" if both conditions are true. If neither condition is true it outputs "*none*." If such a Mealy machine does not exist prove why not.

5. (15 pts) Let X = {*a, b*} and Y = {*1, 2, none, both*}. Construct a Moore machine that outputs a *1* if the input string has 1 more *a*'s than *b*'s or 1 less *a*'s than *b*'s; outputs a *2* if the input string has 2 more or 2 less *a*'s than *b*'s; and outputs "*none*" otherwise. If such a Moore machine does not exist prove why not.

6. (10 pts) Find a grammar that generates $L_1 \cup L_2$ where
   $L_1 = \{ a^n b^{3n} : \ n \geq 0 \}$ and
   $L_2 = \{ a^n b^m : \ n \geq 0, m > n \}$.

7. (15 pts) Let X = {*a, b*} and Y = {*0, 1*} and consider $\beta: X^+ \to Y$ where:

   (a) $\beta(x) = \{$ *1* if the numbers of *a*'s and *b*'s in *x* are equal,
   0 otherwise.

   (b) $\beta(x) = \{$ *1* if *x* contains an odd number of the subsequence *abab*,
   0 otherwise.

   (i) For each of the two functions above, if β is not finite state realizable, prove it; if β is finite state realizable give a transition graph for the corresponding Mealy machine.
   (ii) For each of the two functions above, if β is not finite state realizable, describe the Nerode equivalence classes [x], or equivalently the set of functions $\beta \circ \ell_x$ ($x \in X^*$), for the reduced machine M_β or $M(\beta)$.

8. This problem will not be graded. However, I will go over the problem solution when I cover the homework solutions. So it will be useful to do the problem.

   Minimize the following Mealy machine using the state reduction algorithm discussed in class (that is, find the reduced machine).

Q \ X	0	1
q_0	$q_3/0$	$q_4/0$
q_1	$q_4/0$	$q_3/0$
q_2	$q_1/1$	$q_3/0$
q_3	$q_4/1$	$q_0/0$
q_4	$q_4/1$	$q_1/0$
q_5	$q_1/1$	$q_4/0$