$X = \{0,1\}$ $\quad$ $\beta : X^+ \to Y$ $\quad$ $\beta(x) = \begin{cases} \underline{\underline{1}} \text{ if \# of 1's in } x \text{ greater or equal to \# of 0's.} \\ \underline{\underline{0}} \text{ otherwise} \end{cases}$

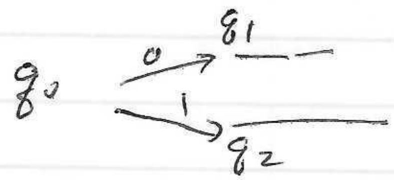
$Y = \{0,1\}$

$\# \text{ 1's} \geq \# \text{ 0's}$

Example

| | | |
|---|---|---|
| | 10110 | output is 1 |
| | 00 111 | output is 1 |
| | 001 | output is 0 |
| | 00000 11111 0 | output is 0. |

Can we build such a finite-state Mealy Machine



realizes above $\beta$ and has a finite # of states

First: What is the intuition?

Second: How do you prove this

$\beta(z)$ is the behavior (starting)

$\beta \circ l_x(z) = \beta(xz)$.

Goal choose a set of behavior functions that are different & an countably infinite number.

$|\{\beta \circ l_x \mid x \in X^*\}|$ in $\underline{\underline{\text{not}}}$ finite.

| $z$ | $\beta(z)$ | $\beta \circ l_0(z)$ | $\beta \circ l_{00}(z)$ | $\beta \circ l_{000}(z)$ | $\beta \circ l_{0000}(z)$ | ... |
|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 0 | 0 | 0 | |
| 11 | 1 | 1 | 1 | 0 | 0 | |
| 111 | 1 | 1 | 1 | 1 | 0 | |
| 1111 | 1 | 1 | 1 | 1 | 1 | |
| ⋮ | | | | | | |

We illustrate that $\exists$ an infinite # of behavior functions for this $\beta : X^+ \to Y$ function. Hence it is not finite state realizable

# State Minimization Algorithm
## Example Moore Machine



First partition on output function $\lambda$.
class 1 $\{0,3\}$ class 2 $\{1,2,4,5\}$

| | Q\X | 0 | 1 |
|---|---|---|---|
| class 1 | 0 | 0 : C1 | 1 : C2 |
| | 3 | 3 : C1 | 4 : C2 |
| class 2 | 1 | 1 : C2 | 2 : C2 |
| | 2 | 2 : C2 | 3 : C1 |
| | 4 | 4 : C2 | 5 : C2 |
| | 5 | 5 : C2 | 0 : C1 |

Note no change to class 1, Cannot be further split at this time
class 2 must be split
$\{1,4\}$ & $\{2,5\}$

| | Q\X | 0 | 1 |
|---|---|---|---|
| class 1 | 0 | 0 : C1 | 1 : C2 |
| | 3 | 3 : C1 | 4 : C2 |
| class 2 | 1 | 1 : C2 | 2 : C3 |
| | 4 | 4 : C2 | 5 : C3 |
| class 3 | 2 | 2 : C3 | 3 : C1 |
| | 5 | 5 : C3 | 0 : C1 |

No change to any classes
Algorithm terminates

Equivalence sets of states are:
$\{0,3\}$, $\{1,4\}$, $\{2,5\}$
Reduced machine has 3 states.