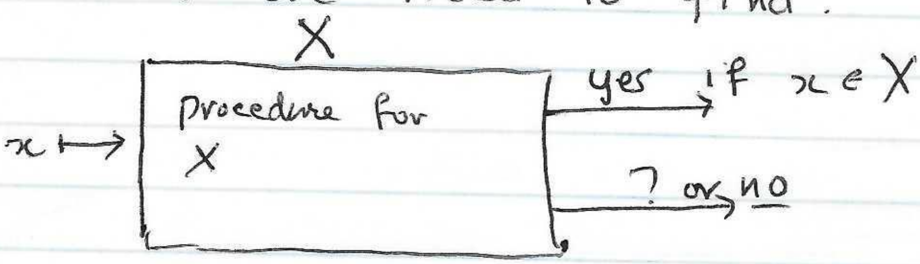
Notes 2

Given procedure to generate a language X we can give a procedure to recognize X.

What do we start with?

$$\left.\begin{array}{ll} 1. & x_1 \\ 2. & x_2 \\ 3. & x_3 \\ 4. & x_4 \\ 5. & x_5 \\ & \vdots \\ & \vdots \end{array}\right\} \text{Generator.}$$

What do we need to find?



The procedure in words: given input x, run generator and if some x_i generated equals x, then output yes.

Example generator of strings (a grammar)

$$
\left.\begin{array}{l}
S \to aSBC \\
S \to aBC \\
CB \to BC \\
aB \to ab \\
bB \to bb \\
bC \to bc \\
cC \to cc
\end{array}\right]
$$

$G = \langle N, T, P, S \rangle$

Start symbol $\doteq$ S

Nonterminals : $\{S, B, C\}$

Terminals : $\{a, b, c\}$

Productions

A string is generated or <u>derived</u> by G if a series of replacements made by use of productions (rules) yields a terminal string.

$$
\begin{aligned}
S &\Rightarrow aSB\underline{C} \Rightarrow aaSBCBC \Rightarrow aaa\underline{BC}BCBC \\
&\Rightarrow aaab\underline{C}CBCBC \\
&\Rightarrow aaa\,bcCBCBC \\
&\Rightarrow aaa\,bccBCBC \\
&\Rightarrow aaa\,bcc\underline{BBC}C \\
&\Rightarrow
\end{aligned}
$$

stuck!

But

$$
\begin{aligned}
S &\Rightarrow aSBC \Rightarrow aaSBCBC \Rightarrow aaaBCBCBC \\
&\Rightarrow aaaBBCCBC \Rightarrow aaaBBCBCC \\
&\Rightarrow aaaBBBCCC \\
&\Rightarrow aaabBBCCC \\
&\Rightarrow aaabbBCCC \\
&\Rightarrow aaabbbCCC \\
&\Rightarrow aaabbbcCC \stackrel{*}{\Rightarrow} aaa\,bbb\,ccc
\end{aligned}
$$

Input - Output Behavior

$X^+ \times Y^+$ we however want to
$(x_1 x_2 x, \; y_1 y_2)$ match the input & output lengths

$X^+ \times Y^+$ & $\ell(x) \in X^+ = \ell(y) \in Y^+$
when $\ell(x)$ is the length of x.

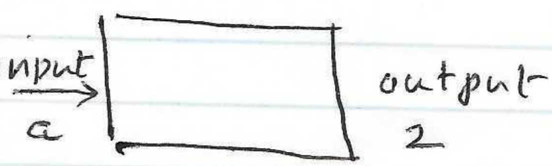
I - O Behavior is a subset of $X^+ \times Y^+$
with the length restriction

R is a subset of $X^+ \times Y^+$.

Example $X \times Y$ $X = \{a, b, c, d\}$
$Y = \{1, 2, 3\}$

$R = \{ (a,1), (a,2), (b,3), (c,1), (b,1) \}$

Viewpoint is



M is the machine
R^M is the complete I-O behavior.

Example $X^+ \times Y^+$ $X = \{0,1\}$ $Y = \{0,1\}$

$R^M = \{ (100, 001), (101, 000),$
$(1001, 1010), (1001, 0110)$
$(1001, 0010), (1001, 1110) \}$
$\circ \circ \circ \circ$

Can be an infinite set of I-O pairs.