Countability

Set S. Size of set S written as $|S|$.
For finite sets size is pretty clear

$A = \{a, b, d\}$ $|A| = 3$.

$B = \{x, y, z\}$ $|B| = 3$.
Are A & B the same size?
Why?

$$
\begin{array}{ccc}
f: \underline{A} & \rightarrow & \underline{B} \\
a & \mapsto & x \\
b & \mapsto & y \\
d & \mapsto & z
\end{array}
$$

f is a 1 to 1 onto function.

Two sets are the same size if there is a 1-1 onto function between the sets.

Cardinality of the integers : $\{1, 2, 3, \ldots\}$
Infinite but <u>countable</u>.
Def $\leftarrow$ $\exists$ a 1-1 match with the integers

Size of Integers $\aleph_0$ Aleph nought.
Countably Infinite

Can show that the size of even numbers is also $\aleph_0$.

Countability

$I \to E$

$f(x) = 2x$

$$\begin{array}{cccc} 1 & 2 & 3 & \\ \downarrow & \downarrow & \downarrow & \cdots \\ 2 & 4 & 6 & \end{array}$$

But note that the even numbers are also a subset of the Integers.

What about the real numbers? R.

There are also infinite but <u>bigger</u> than the size of the integers, why?

$|R| \geq |I|$ Obvious because $I \subset R$

But there can be no 1-1 match between reals & integers. Why – diagonalization.
Hence we say $|R|$ is $\aleph_1$?
(Assuming the continuum hypothesis) $\equiv$
Given any set S, size of 2^S, the power set is strictly greater than S.

Note: need to assume some model of sets such as Zermelo Fraenkel (ZF) or ZF with axiom of choice (ZFC)

Diagonalization

Can use instead of all real numbers just the numbers [0, 1] in this interval
For convenience using a binary expansion.
Assumption: Any real # can be written as a countable expansion of the integers.

. 0 1 1 0 1 0 1 1 1 1 0 ....

Suppose there was a 1-1 mapping of integers to the reals in [0, 1].
List all the reals in some way as the matching.

$x_1 : . \boxed{0} 1 0 1 1 1 0 1 \ldots$

$x_2 : . 1 \boxed{0} 1 1 0 1 0 1 1 1 \ldots$

$x_3 : . 0 0 \boxed{1} 1$

$x_4 : \quad \vdots \quad \boxed{0}$

$\vdots$

Look at . 1 1 0 1 ....
This number is <u>not</u> in the countable <u>set</u>!

Languages & Recognition

Alphabet V. → finite set.
All strings or sentences over V is V^*
A Language is a subset of V^*

$\{a, b, c\} = V$

Example of strings over V.

abaab
bcba

How about ababcb ..... × No

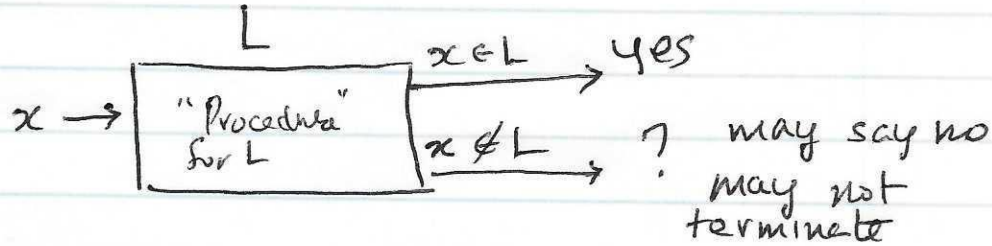
Example of a Language over V.

$\{ab, abab, ababab, \ldots\} = L \subset V^*$

We want to recognize languages as follows:

A Procedure:

Input string x:



An Algorithm