

your name: _____

This is a closed-book examination. You are not allowed to use any notes or other references during the exam. You will have 75 minutes to answer 7 out of the 8 questions. (I will only grade the first 7 answers.) Except where I have provided grids for Gantt charts, you should answer on the blank paper I provide you. Each numbered question is worth 15 points. No partial credit will be given for a wrong answer unless you explain how you obtained it and I can see from the explanation where you went astray.

Answers shown in italic font with green highlighting.

1. What is the utilization bound for Rate Monotonic scheduling? On what assumptions about the task set, scheduling algorithm, and number of processors does this depend? (Use separate paper.)

A task set is schedulable if (not only if) $U_{sum} \leq n(2^{\frac{1}{n}} - 1)$, where A task set is schedulable if (not only if)

$$U_{sum} \leq n(2^{\frac{1}{n}} - 1), \text{ where } U_{sum} = \sum_{i=1}^n \frac{e_i}{p_i}.$$

Assumptions:

- 1. preemptive fixed-priority scheduling*
- 2. periodic or sporadic tasks*
- 3. deadline = period (implicit deadlines)*
- 4. single processor*
- 5. independent tasks*

2. What is the utilization bound for Earliest-Deadline-First scheduling? On what assumptions about the task set, scheduling algorithm and task set does this depend? (Use separate paper.)

A task set is schedulable if and only if $U_{sum} \leq 1$, where $U_{sum} = \sum_{i=1}^n \frac{e_i}{p_i}$

Assumptions:

- 6. preemptive scheduling with higher priority to jobs with earlier absolute deadline*
- 7. periodic or sporadic tasks*
- 8. deadline = period (implicit deadlines)*
- 9. single processor*
- 10. independent tasks*

This is the utilization bound. There is also a density bound, which is not only-if, for constrained-deadline tasks.

The following questions refer to the sporadic task set described the table on the right. Deadlines are implicit; that is, the relative deadline of each task is equal to its period.

task	period	exec. time
T1	7	3
T2	8	4

3. Suppose that the execution time of task T2 is increased from 4 to 4.5. Assuming worst-case phasing and job arrivals, is the task set schedulable on one processor by preemptive EDF scheduling? Justify your answer. (Use separate paper.)

$U_{sum} = \frac{3}{7} + \frac{4.5}{8} = \frac{48+63}{112} = \frac{111}{112} < 1$, so the task set is schedulable.

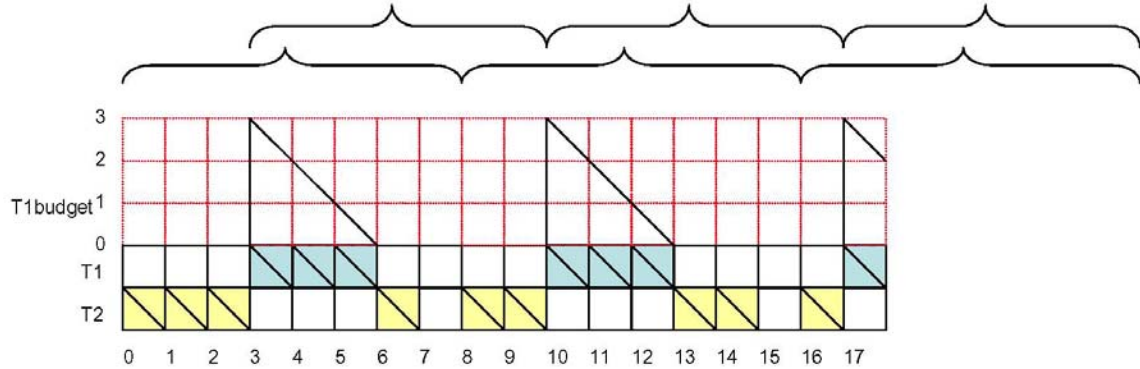
4. Under the same other assumptions as the question immediately above, but with RM scheduling, what is the worst-case response time of T2? Justify your answer. (Use separate paper.)

$$t_0 = 4.5 + 3 = 7.5$$

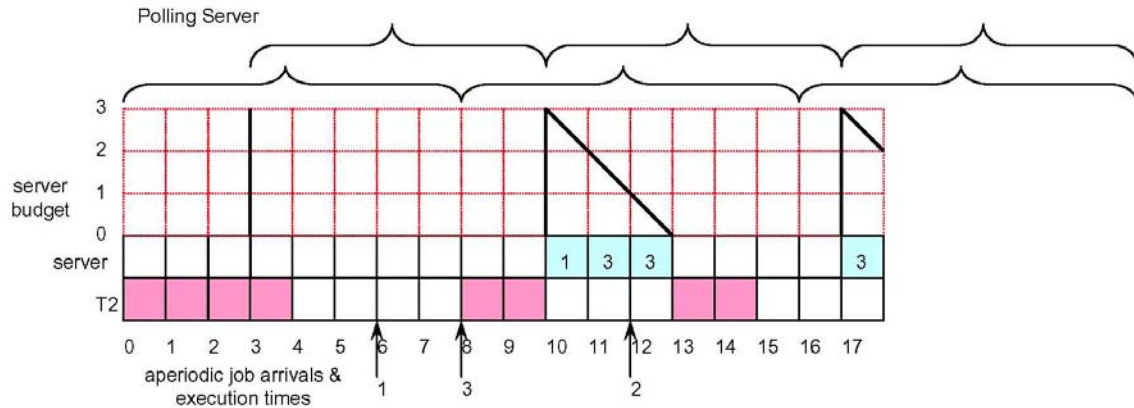
$$t_1 = 4.5 + \left\lceil \frac{7.5}{7} \right\rceil \cdot 3 = 10.5$$

$$t_2 = 4.5 + \left\lceil \frac{10.5}{7} \right\rceil \cdot 3 = 10.5$$

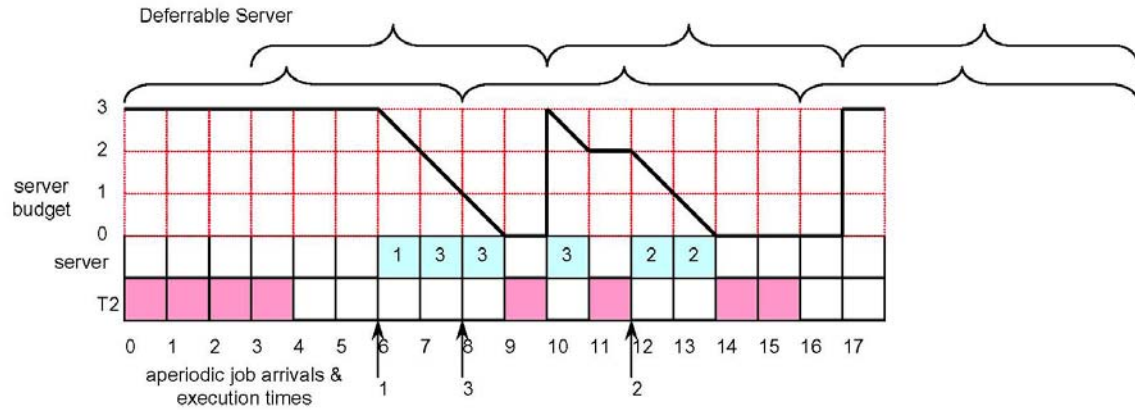
The figure below shows a Rate Monotonic schedule for this task set, assuming jobs of the tasks arrive periodically, with scheduling windows (phasing) indicated by the braces. It is provided to illustrate the kind of answer you are to provide for the following questions.



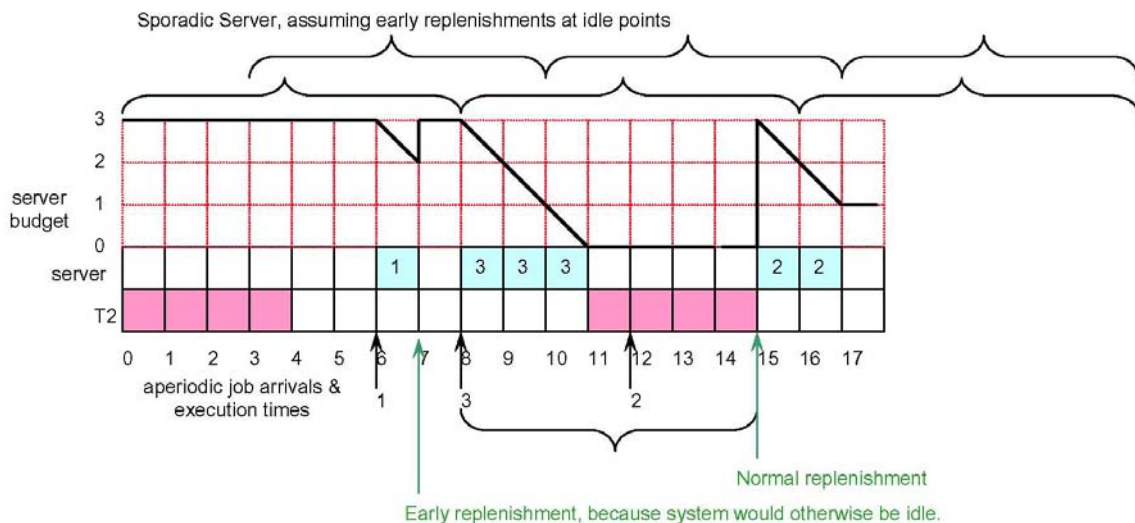
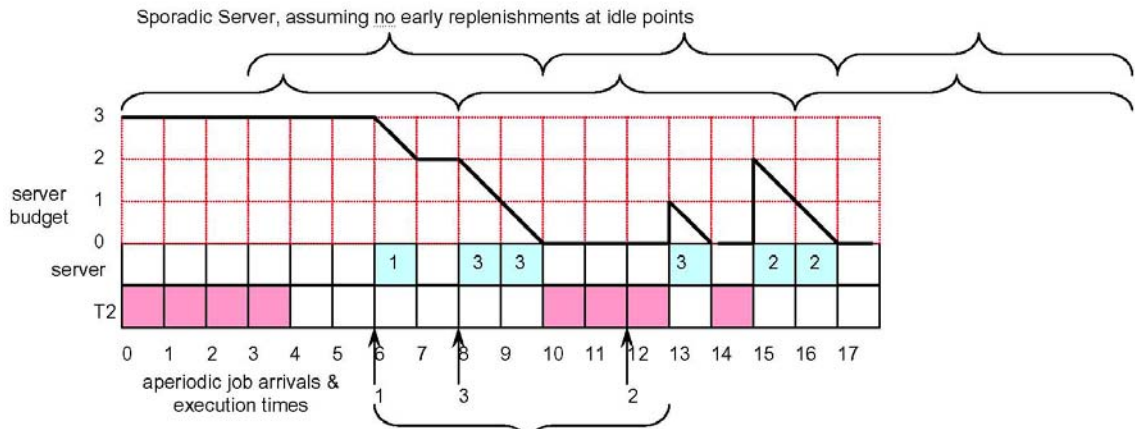
5. Suppose task T1 is converted to a Polling Server with the budget (3) and period (7) given in the table. Assume fixed-priority preemptive scheduling with RM priorities and the phasings shown by the braces. Fill in the schedule below, showing the execution of T2 and the server, assuming three aperiodic jobs arrive at the times indicated by the arrows. The execution times of the aperiodic jobs are 1, 3, and 2, respectively. On the upper part of the grid given on the following page, show the graph of the server budget.



4. Now suppose the server is scheduled as a Deferrable Server with the same budget and period, still assuming fixed-priority preemptive scheduling with RM priorities, the same phasings, and the same set of aperiodic jobs.. Fill in the schedule below, showing the execution of T2 and the server, using the upper part of the grid to graph the server budget. (15 pts)



5. Now suppose the server is scheduled as a Sporadic Server with the same budget and period, still assuming fixed-priority preemptive scheduling with RM priorities, the same phasings, and the same set of aperiodic jobs.. Fill in the schedule below, showing the execution of T2 and the server, using the upper part of the grid to graph the server budget. Assume the SpSL version of the Sporadic Server algorithm, or if you don't know that one (deduction of 3 pts) specify which other version you are following.

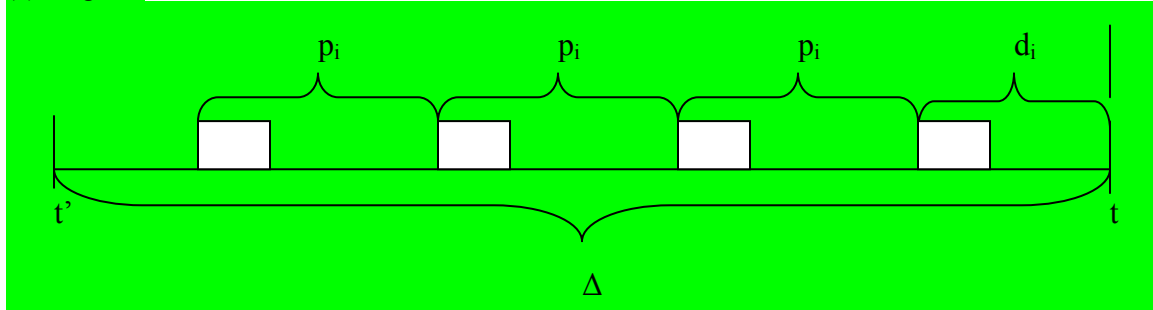


7. Explain the derivation of the utilization bound result for EDF scheduling. Your answer must include at least: (1) a definition of the maximal busy interval preceding the deadline of a job; (2) a diagram showing

the pattern of arrivals that generates the worst-case interference caused for a task T_k from a single other task T_i in an interval leading to the deadline of a job of T_k ; (3) and explanation of how the utilization bound is derived from this worst case. (Only answer this if you have skipped one of the above questions.)

(1) Let t be the deadline of a job. The maximal busy interval preceding t is the longest interval $[t', t)$ such that the processor is continually busy during this interval, executing jobs with release times $\leq t'$ and with deadlines $\geq t$.

(2) Diagram:



(3) To miss the deadline, the execution times of tasks in this interval must sum to $> t - t' = \Delta$, and so

$$\sum_{i=1}^n e_i \left\lceil \frac{\Delta - d_i}{p_i} \right\rceil + e_i > \Delta$$

$$\sum_{i=1}^n u_i \frac{\Delta + p_i - d_i}{\Delta} = \frac{\sum_{i=1}^n e_i \frac{\Delta - d_i + p_i}{p_i}}{\Delta} \geq \frac{\sum_{i=1}^n e_i \left\lceil \frac{\Delta - d_i + p_i}{p_i} \right\rceil}{\Delta} > 1$$

$$u_{sum} = \sum_{i=1}^n u_i \geq \sum_{i=1}^n u_i \left(1 + \frac{p_i - d_i}{\Delta} \right) > 1$$

The last step follows from $p_i \geq d_i$.