COT 5405: Advanced Algorithms
Fall 2011

Assignment 1

Due: 5pm, 25 Oct 2011

1. (20 points) Give the dual of the following linear program.

Minimize $2x_1 - 4 x_2$
Subject to:
$3 x_1 + 2 x_2 \geq 4$
$2 x_1 - x_2 \geq 6$
$4 x_1 - 2 x_2 \geq -2$
$-3 x_1 - 5 x_2 \geq -3$
$x_1, x_2 \geq 0$

Answer:

maximize $4y_1 + 6y_2 - 2y_3 - 3y_4$

Subject to:
$3y_1 + 2y_2 + 4y_3 - 3y_4 \leq 2$
$2y_1 - y_2 - 2y_3 - 5y_4 \leq -4$
$y_1, y_2, y_3, y_4 \geq 0$

2. (20 points) Given the following instance of Knapsack: *profits* (4, 20, 12, 12, 2), *sizes* (2, 7, 4, 4, 1), and *capacity* 9, find a factor *1/2* approximation yielded by the FTPAS we discussed in class. Show all the steps in the algorithm.

Answer:

K = 0.5*20/5 = 2
p' = (2, 10, 6, 6, 1)

	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**
1	0	∞	2	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
2	0	∞	2	∞	∞	∞	∞	∞	∞	∞	7	∞	9	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
3	0	∞	2	∞	∞	∞	4	∞	6	∞	7	∞	9	∞	∞	∞	11	∞	13	∞	∞	∞	∞	∞	∞	∞
4	0	∞	2	∞	∞	∞	4	∞	6	∞	7	∞	8	∞	10	∞	11	∞	13	∞	∞	∞	15	∞	17	∞
5	0	1	2	3	∞	∞	4	5	6	7	7	8	8	**9**	10	11	11	12	13	14	∞	∞	15	16	17	18

The solution corresponds to profit' = 14 with elements {3, 4, 5} with actual profit 26.

3. (20 points) Given the following instance of set cover: *sets* {a, b}, {a, c, d}, {b, d e}, and {a, b, e}, with *costs* 2, 4, 3, and 3 respectively, find the solution using the primal-dual

algorithm discussed in class. Pick the *y*s in alphabetical order. Show all the steps in the algorithm.

Answer:

S' = {}, C' = {}

Step 1:
$x_1 = 0$ or $y_a + y_b = 2$
$x_2 = 0$ or $y_a + y_c + y_d = 4$
$x_3 = 0$ or $y_b + y_d + y_e = 3$
$x_4 = 0$ or $y_a + y_b + y_e = 3$

$y_a = 2$. S' = {s_1}, C'={a, b}

Step 2:
$x_2 = 0$ or $y_c + y_d = 2$
$x_3 = 0$ or $y_d + y_e = 3$
$x_4 = 0$ or $y_e = 1$

$y_c = 2$. S' = {s_1, s_2}, C'={a, b, c, d}

Step 3:
$x_3 = 0$ or $y_e = 3$
$x_4 = 0$ or $y_e = 1$

$y_e = 1$. S' = {s_1, s_2, s_4}, C'={a, b, c, d, e}

Cost = 9

4. (20 points) Formulate the following *Minimum Edge Dominating Set* problem as an integer linear program, and also give its relaxation. *Minimum Edge Dominating Set*: Given a graph *G = (V, E)*, find a subset of edges, *E'*, of smallest cardinality, such that if $e_1 \in E - E'$, then there is an $e_2 \in E'$ such that e_1 and e_2 are adjacent.

Answer:

ILP
Minimize $\sum_{e \in E} x_e$
Subject to:
$\sum_{e' \in E:\, e \cap e' \neq \emptyset} x_{e'} \geq 1 \ \forall e \in E$
$x_e \in \{0,1\} \ \forall e \in E$

Relaxation
Minimize $\sum_{e \in E} x_e$
Subject to:
$\sum_{e' \in E:\, e \cap e' \neq \emptyset} x_{e'} \geq 1 \ \forall e \in E$
$x_e \geq 0 \ \forall e \in E$

5. (20 points) Show that the following approximation algorithm for set cover has an approximation factor of $|U|$, and show that this bound is tight. *Note: In this problem, we define the cost of a set as the sum of the cost of each of its elements.*

```
C := { }
while C ≠ U
        Let s be a set of smallest cost which contains some uncovered element
        C := C ∪ s
Output the sets picked
```

Answer:

$OPT \geq \sum_{e \in U} c_e$, where c_e is the cost of element e.
The approximation algorithm picks at most $|U|$ sets and each has cost at most $\sum_{e \in U} c_e$.
Therefore $APPROX \leq |U|\ OPT$

Tightness: Let U = {a, b} with costs {K, 1} and the sets be {a}, and{a, b}. The algorithm will first choose the first set and then the second, with total cost 2K+1, while the optimum is to choose the second set with total cost K. As K approaches infinity, APPROX/OPT approaches 2, which is |U|.