COT 5405: Advanced Algorithms
Fall 2011

Assignment 1

Due: 5pm, 25 Oct 2011

1. (20 points) Give the dual of the following linear program.

Minimize $2x_1 - 4\,x_2$
Subject to:
$3\,x_1 + 2\,x_2 \geq 4$
$2\,x_1 - x_2 \geq 6$
$4\,x_1 - 2\,x_2 \geq -2$
$-3\,x_1 - 5\,x_2 \geq -3$
$x_1, x_2 \geq 0$

2. (20 points) Given the following instance of Knapsack: *profits* (4, 20, 12, 12, 2), *sizes* (2, 7, 4, 4, 1), and *capacity* 9, find a factor *1/2* approximation yielded by the FTPAS we discussed in class. Show all the steps in the algorithm.

3. (20 points) Given the following instance of set cover: *sets* {a, b}, {a, c, d}, {b, d e}, and {a, b, e}, with *costs* 2, 4, 3, and 3 respectively, find the solution using the primal-dual algorithm discussed in class. Pick the *y*s in alphabetical order. Show all the steps in the algorithm.

4. (20 points) Formulate the following *Minimum Edge Dominating Set* problem as an integer linear program, and also give its relaxation. *Minimum Edge Dominating Set*: Given a graph $G = (V, E)$, find a subset of edges, E', of smallest cardinality, such that if $e_1 \in E - E'$, then there is an $e_2 \in E'$ such that e_1 and e_2 are adjacent.

5. (20 points) Show that the following approximation algorithm for set cover has an approximation factor of $|U|$, and show that this bound is tight. *Note: In this problem, we define the cost of a set as the sum of the cost of each of its elements.*

$C := \{\ \}$
while $C \neq U$
    Let s be a set of smallest cost which contains some uncovered element
    $C := C \cup s$
Output the sets picked