COT 5405: Advanced Algorithms
Fall 2006

Assignment 1

Due: 5pm, 29 Sep 2006

1. (20 points) Given the following instance of set cover: *sets* {a, b}, {a, c, d}, {b, d e}, and {a, b, e}, with *costs* 3, 2, 3, and 4 respectively, find the solution using the greedy algorithm discussed in class. Show all the steps in the algorithm.

2. (20 points) Given the following instance of Knapsack: *profits* (5, 5, 7, 10, 4), *sizes* (2, 2, 3, 6, 1), and *capacity* 7, find an approximate solution yielded by the FTPAS we discussed in class. Take $\varepsilon = 2$, even though you would not do this in a real application. Show all the steps in the algorithm.

3. (20 points) Given the following instance of set cover: *sets* {a, b}, {a, c, d}, {b, d e}, and {a, b, e}, with *costs* 2, 4, 3, and 3 respectively, find the solution using the primal-dual algorithm discussed in class. Pick the *y*s in alphabetical order. Show all the steps in the algorithm. *Note that the costs here are different from those in problem 1.*

4. (20 points) Formulate the *minimum dominating set* problem as an integer linear program. This problem is defined as follows. *Given a graph $G = (V, E)$, find a dominating set of smallest cardinality*. A set $V' \subseteq V$ is a dominating set if for each $u \in V\text{-}V'$, there exists a $v \in V'$ such that $\{u, v\} \in E$.

5. (20 points) (From **AA**) Consider the following factor *2* approximation algorithm for *cardinality vertex cover* on a connected graph. Find a depth first search tree in the given graph, *G*, and output the set, say *S*, of all the non-leaf vertices of this tree. Show that *S* is a vertex cover for *G* and $|S| \leq 2\,OPT$. *Hint:* Show that *G* has a matching of size at least $|S|/2$.