Rate Monotonic Analysis

Introduction

Periodic tasks

Extending basic theory

Synchronization and priority inversion

Aperiodic servers

Case study: BSY-1 Trainer

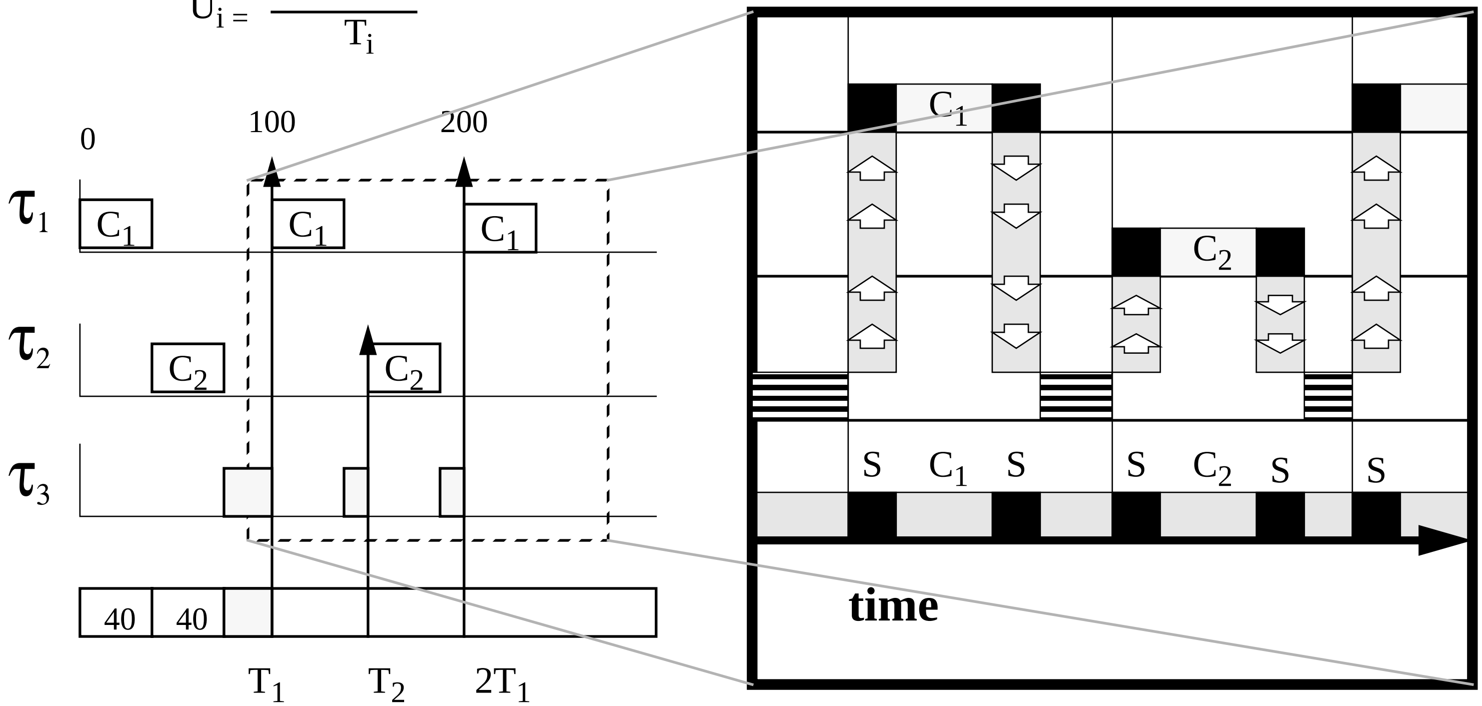
Extensions to Basic Theory

This section extends the schedulability tests to address

- **nonzero task switching times**
- **preperiod deadlines**
- **interrupts and non-rate-monotonic priorities**

Modeling Task Switching as Execution Time

$$U_i = \frac{C_i + 2S}{T_i}$$



Two scheduling actions per task
(start of period and end of period)

Modeling Preperiod Deadlines

Suppose task τ, with compute time *C* and period *T*, has a preperiod deadline *D* (i.e. *D* < *T*).

Compare total utilization to modified bound:

$$U_{total} = \frac{C_1}{T_1} + \ldots + \frac{C_n}{T_n} \le U(n, \Delta_i)$$

where Δ_i is the ratio of D_i to T_i .

$$U(n, \Delta_i) = \begin{cases} n\left((2\Delta_i)^{1/n} - 1\right) + 1 - \Delta_i, & \frac{1}{2} < \Delta_i \le 1.0 \\ \Delta_i, & \Delta_i \le \frac{1}{2} \end{cases}$$

Schedulability with Interrupts

Interrupt processing can be inconsistent with rate monotonic priority assignment.

- **interrupt handler executes with high priority despite its period**
- **interrupt processing may delay execution of tasks with shorter periods**

Effects of interrupt processing must be taken into account in schedulability model.

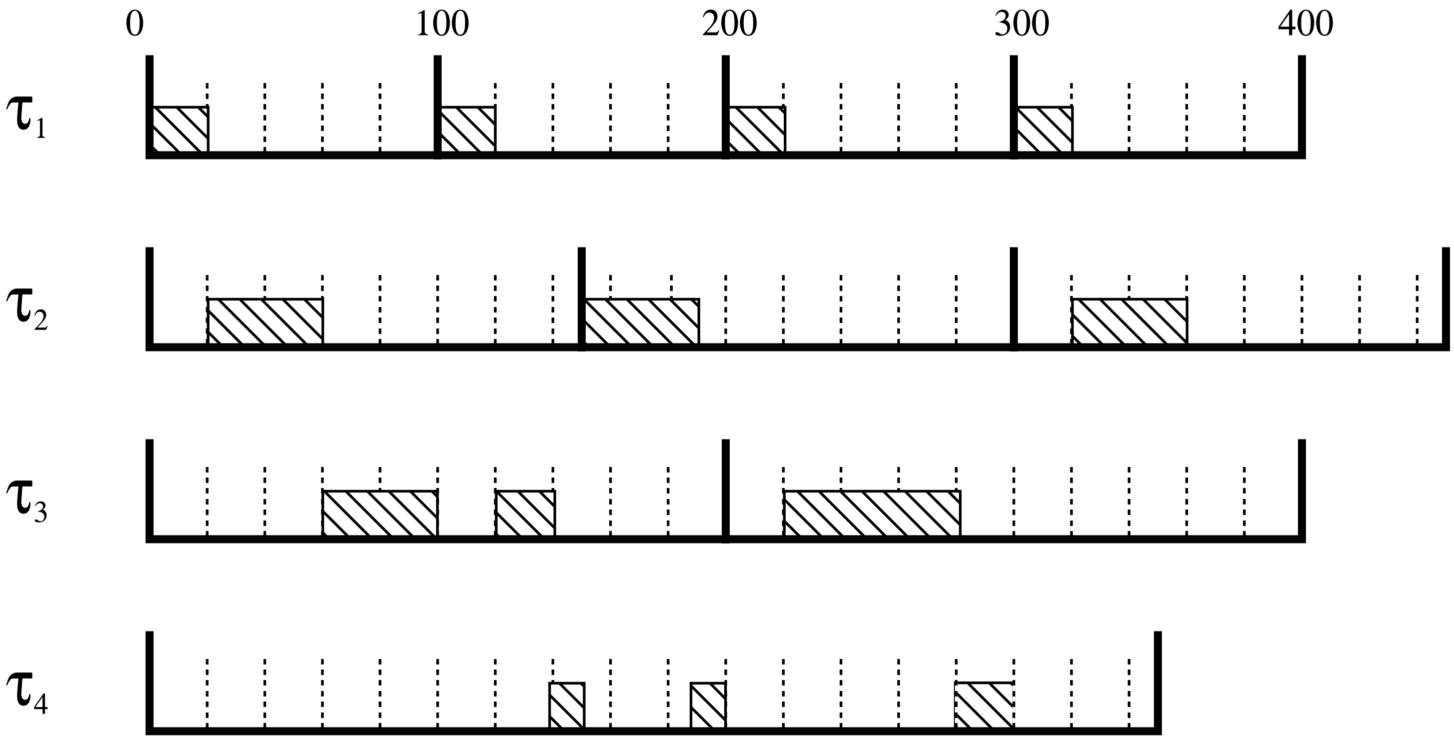
Question is: how to do that?

Example: Determining Schedulability with Interrupts

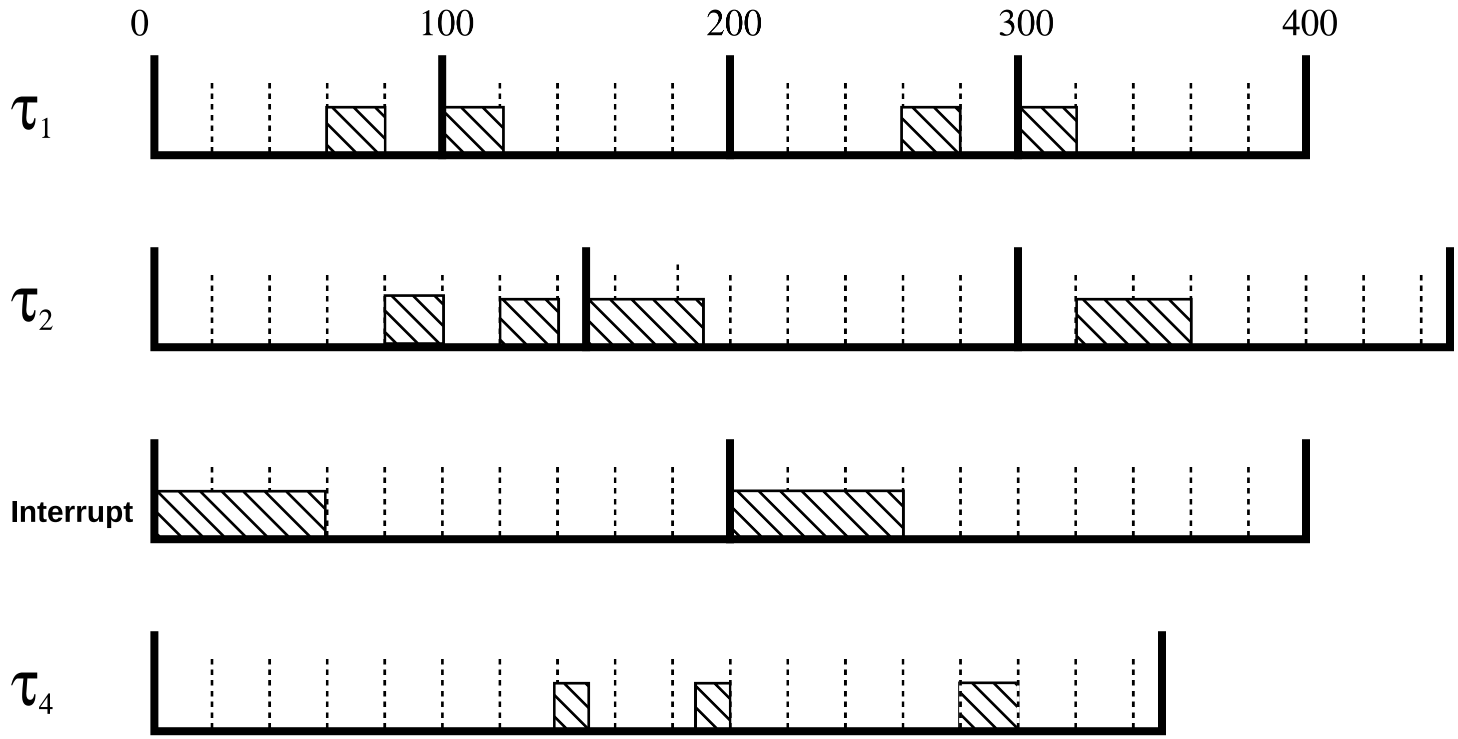
	C	*T*	*U*
Task τ_1:	**20**	**100**	**0.200**
Task τ_2:	**40**	**150**	**0.267**
Task τ_3:	**60**	**200**	**0.300**
Task τ_4:	**40**	**350**	**0.115**

τ_3 is an interrupt handler

Example: Execution with Rate Monotonic Priorities



Example: Execution with an Interrupt Priority



Resulting Table for Example

Task (i)	Period (T)	Execution Time (C)	Priority (P)	Deadline (D)
τ_3	200	60	HW	200
τ_1	100	20	High	100
τ_2	150	40	Medium	150
τ_4	350	40	Low	350

UB Test with Interrupt Priority

Test is applied to each task.

Determine effective utilization (f_i) of each task *i* using

$$f_i = \boxed{\sum_{j \in Hn} \frac{C_j}{T_j}} + \boxed{\frac{C_i}{T_i}} + \boxed{\frac{1}{T_i} \sum_{k \in H1} C_k}$$

Preemption from tasks that can hit more than once (with period less than D_i)

Execution of task under test

Preemption from tasks that can hit only once (with period greater than D_i)

Compare effective utilization against bound, U(n).

- **n = num(*Hn*) + 1**
- **num(*Hn*) = the number of tasks in the set *Hn***

UB Test with Interrupt Priority: τ_3

For τ_3, no tasks have a higher priority: *H* = *Hn* = *H1* = { }.

$$f_3 \quad = \quad 0 + \frac{C_3}{T_3} + 0 \quad \leq U(1)$$

Note that utilization bound is U(1): num(*Hn*) = 0.

Plugging in numbers:

$$f_3 = \frac{C_3}{T_3} = \frac{60}{200} = 0.3 < 1.0$$

UB Test with Interrupt Priority: τ_1

To τ_1, τ_3 has higher priority: *H* = {τ_3}; *Hn* = { }; *H1* = {τ_3}.

$$f_1 = 0 + \frac{C_1}{T_1} + \frac{1}{T_1} \sum_{k=3} C_k \le U(1)$$

Note that utilization bound is U(1): num(*Hn*) = 0.

Plugging in the numbers:

$$f_1 = \frac{C_1}{T_1} + \frac{C_3}{T_1} = \frac{20}{100} + \frac{60}{100} = 0.800 < 1.0$$

UB Test with Interrupt Priority: τ_2

To τ_2: *H* = {τ_1,τ_3}; *Hn* = {τ_1}; *H1* = {τ_3}.

$$f_2 = \sum_{j=1} \frac{C_j}{T_j} + \frac{C_2}{T_2} + \frac{1}{T_2} \sum_{k=3} C_k \quad \le U(2)$$

Note that utilization bound is U(2): num(*Hn*) = 1.

Plugging in the numbers:

$$f_2 = \frac{C_1}{T_1} + \frac{C_2}{T_2} + \frac{C_3}{T_2} = \frac{20}{100} + \frac{40}{150} + \frac{60}{150} = 0.867 \quad > 0.828$$

UB Test with Interrupt Priority: τ_4

To τ_2: *H* = {τ_1,τ_2,τ_3}; *Hn* = {τ_1,τ_2,τ_3}; *H1* = { }.

$$f_4 = \sum_{j = 1, 2, 3} \frac{C_j}{T_j} + \frac{C_4}{T_4} + 0 \quad \le U(4)$$

Note that utilization bound is U(4): num(*Hn*) = 3.

Plugging in the numbers:

$$f_4 = \frac{C_1}{T_1} + \frac{C_2}{T_2} + \frac{C_3}{T_3} + \frac{C_4}{T_4}$$

$$= \frac{20}{100} + \frac{40}{150} + \frac{60}{200} + \frac{40}{350} = 0.882 \quad > 0.756$$

Exercise: Schedulability with Interrupts

Given the following tasks:

Task (i)	Period (T)	Execution Time (C)	Priority (P)	Deadline (D)
τ_{int}	6	2	HW	6
τ_1	4	1	High	3
τ_2	10	1	Low	10

Use the UB test to determine which tasks are schedulable.

Solution: Schedulability with Interrupts

$$\frac{C_{int}}{T_{int}} \le U(1) \qquad 0.334 < 1.0$$

{H1}

$$\frac{C_1}{T_1} + \frac{C_{int}}{T_1} \le U(1, .75) \qquad 0.250 + 0.500 = 0.750 = U(1, .75)$$

{Hn}

$$\frac{C_{int}}{T_{int}} + \frac{C_1}{T_1} + \frac{C_2}{T_2} \le U(3)$$

$$0.334 + 0.250 + 0.100 = 0.684 < 0.779$$

Basic Theory: Where Are We?

We have shown how to handle

- **task context switching time: include 2*S* within *C***
- **preperiod deadlines: change bound to U(n, Δ_i)**
- **non-rate-montonic priority assignments**

We still must address

- **task interactions**
- **aperiodic tasks**

We still assume

- **single processor**
- **priority-based scheduling**
- **tasks do not suspend themselves**

Other Important Issues

Mode change

Multiprocessor systems

Priority granularity

Overload

Spare capacity assessment

Distributed systems

Post-period deadlines